Ploče opterećene na savijanje – izrazi za sile

$$M_x = -k \cdot \left[\frac{\partial^2 w}{\partial x^2} + \nu \cdot \frac{\partial^2 w}{\partial y^2} \right]$$

$$M_y = -k \cdot \left[\frac{\partial^2 w}{\partial y^2} + \nu \cdot \frac{\partial^2 w}{\partial x^2} \right]$$

$$M_{xy} = -k \cdot (1-\nu) \cdot \frac{\partial^2 w}{\partial x \cdot \partial y}$$

$$T_x = -k \cdot \left[\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \cdot \partial y^2} \right]$$

$$T_y = -k \cdot \left[\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \cdot \partial y} \right]$$

$$\overline{T_x} = -k \cdot \left[\frac{\partial^3 w}{\partial x^3} + (2-\nu) \cdot \frac{\partial^3 w}{\partial x \cdot \partial y^2} \right]$$

$$\overline{T_y} = -k \cdot \left[\frac{\partial^3 w}{\partial y^3} + (2-\nu) \cdot \frac{\partial^3 w}{\partial y \cdot \partial x^2} \right]$$

$$k = \frac{E \cdot h^3}{12(1-\nu^2)}$$

Navier-ovo rešenje

$$w(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cdot \sin\frac{m \cdot \pi \cdot x}{a} \cdot \sin\frac{n \cdot \pi \cdot y}{b}$$

$$A_{mn} = \frac{Z_{mn}}{k \cdot \pi^4 \cdot \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}$$

$$Z_{mn} = \frac{4}{a \cdot b} \cdot \int_x \int_y Z(x,y) \cdot \sin\frac{m \cdot \pi \cdot x}{a} \cdot \sin\frac{n \cdot \pi \cdot y}{b} \cdot dx \cdot dy$$

rešenje za jednako podeljeno opterećenje na celoj ploči

$$A_{mn} = \frac{16 \cdot Z_0}{k \cdot \pi^6 \cdot \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}$$

$$w(x,y) = \frac{16 \cdot Z_0}{k \cdot \pi^6} \cdot \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{m \cdot n \cdot \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \cdot \sin\frac{m \cdot \pi \cdot x}{a} \cdot \sin\frac{n \cdot \pi \cdot y}{b}$$

rešenje za jednako podeljeno opterećenje na pravougaoniku $2c \times 2d$

$$w(x,y) = \frac{16 \cdot Z_0}{k \cdot \pi^6} \cdot \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin\frac{m \cdot \pi \cdot u}{a} \cdot \sin\frac{m \cdot \pi \cdot c}{a} \cdot \sin\frac{n \cdot \pi \cdot v}{b} \cdot \sin\frac{n \cdot \pi \cdot d}{b}}{m \cdot n \cdot \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \cdot$$

$$\cdot \sin\frac{m \cdot \pi \cdot x}{a} \cdot \sin\frac{n \cdot \pi \cdot y}{b}$$

rešenje za koncentrisanu silu u tački (u, v)

$$w(x,y) = \frac{4 \cdot P}{k \cdot a \cdot b \cdot \pi^4} \cdot \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin\frac{m \cdot \pi \cdot u}{a} \cdot \sin\frac{n \cdot \pi \cdot v}{b}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \cdot \sin\frac{m \cdot \pi \cdot x}{a} \cdot \sin\frac{n \cdot \pi \cdot y}{b}$$

M. Levy – jevo rešenje

$$w = w_h + w_p$$

$$w_h = \sum_{n=1}^{\infty}\left[\left(A_n + \frac{n\cdot\pi\cdot y}{a}\cdot B_n\right)\cdot ch\frac{n\cdot\pi\cdot y}{a} + \left(C_n + \frac{n\cdot\pi\cdot y}{a}\cdot D_n\right)\cdot sh\frac{n\cdot\pi\cdot y}{a}\right]\cdot sin\frac{n\cdot\pi\cdot x}{a}$$

$$w_p = \sum_{n=1}^{\infty} w_n \cdot sin\frac{n\cdot\pi\cdot x}{a} \qquad w_n = \frac{Z_n}{k}\cdot\left(\frac{a}{n\cdot\pi}\right)^4$$

$$Z = \sum_{n=1}^{\infty} Z_n \cdot \sin\frac{n\cdot\pi\cdot x}{a} \qquad Z_n = \frac{2}{a}\cdot\int_x Z(x)\cdot\sin\frac{n\cdot\pi\cdot x}{a}dx$$

$$w_{h,sim} = \sum_{n=1}^{\infty}\left(A_n\cdot ch\frac{n\cdot\pi\cdot y}{a} + D_n\cdot\frac{n\cdot\pi\cdot y}{a}\cdot sh\frac{n\cdot\pi\cdot y}{a}\right)\cdot sin\frac{n\cdot\pi\cdot x}{a}$$

$$w_{h,ant} = \sum_{n=1}^{\infty}\left(B_n\cdot\frac{n\cdot\pi\cdot y}{a}\cdot ch\frac{n\cdot\pi\cdot y}{a} + C_n\cdot sh\frac{n\cdot\pi\cdot y}{a}\right)\cdot sin\frac{n\cdot\pi\cdot x}{a}$$

rešenje za slobodno oslonjenu ploču na konturi $y=\pm b/2$ usled jednako podeljenog opterećenja

$$A_n = -\frac{1}{n^5}\cdot\frac{\alpha_n\cdot th\alpha_n + 2}{2\cdot ch\alpha_n}\cdot S \qquad D_n = \frac{1}{n^5}\cdot\frac{S}{2\cdot ch\alpha_n}$$

$$S = 4\cdot\frac{Z_0\cdot a^4}{\pi^5\cdot k} \qquad \alpha_n = \frac{n\cdot\pi\cdot b}{2a}$$

rešenje za uklještenu ploču na konturi $y=\pm b/2$ usled jednako podeljenog opterećenja

$$A_n = -\frac{1}{n^5}\cdot\frac{\alpha_n\cdot ch\alpha_n + sh\alpha_n}{\alpha_n + sh\alpha_n\cdot ch\alpha_n}\cdot S \qquad D_n = \frac{1}{n^5}\cdot\frac{sh\alpha_n}{\alpha_n + sh\alpha_n\cdot ch\alpha_n}\cdot S$$

$$S = 4\cdot\frac{Z_0\cdot a^4}{\pi^5\cdot k} \qquad \alpha_n = \frac{n\cdot\pi\cdot b}{2a}$$

Metoda konačnih razlika – diferencna metoda

$$\alpha = \frac{\Delta y}{\Delta x}$$

$$w_k \cdot \left[6 \cdot \left(\alpha^2 + \frac{1}{\alpha^2}\right) + 8\right] - 4 \cdot \left[(1+\alpha^2) \cdot (w_{k+1} + w_{k-1}) + \left(1 + \frac{1}{\alpha^2}\right) \cdot (w_l + w_i)\right] +$$

$$+ 2 \cdot (w_{i-1} + w_{l-1} + w_{i+1} + w_{l+1}) + \alpha^2 \cdot (w_{k+2} + w_{k-2}) + \frac{1}{\alpha^2} \cdot (w_m + w_h) = \frac{Z_k \cdot \alpha^2 \cdot \Delta x^4}{k}$$

$$\alpha = 1$$

$$20 \cdot w_k - 8 \cdot (w_{k+1} + w_{k-1} + w_l + w_i) + 2 \cdot (w_{i-1} + w_{l-1} + w_{i+1} + w_{l+1}) +$$

$$+ (w_{k+2} + w_{k-2} + w_m + w_h) = \frac{Z_k \cdot \alpha^2 \cdot \Delta x^4}{k}$$

$$M_{x,k} = \frac{k}{\Delta x^2} \cdot \left[-w_{k+1} + 2 \cdot w_k - w_{k-1} + \frac{\nu}{\alpha^2} \cdot (-w_l + 2 \cdot w_k - w_i)\right]$$

$$M_{y,k} = \frac{k}{\Delta y^2} \cdot \left[-w_l + 2 \cdot w_k - w_i + \nu \cdot \alpha^2 \cdot (-w_{k+1} + 2 \cdot w_k - w_{k-1})\right]$$

$$M_{xy,k} = \frac{k}{4 \cdot \Delta x^2 \cdot \alpha} \cdot (1-\nu) \cdot [-w_{l+1} + w_{l-1} + w_{i+1} - w_{i-1}]$$

$$T_{x,k} = \frac{k}{2 \cdot \Delta x^3} \cdot \left[w_{k-2} + 2 \cdot \left(1 + \frac{1}{\alpha^2}\right) \cdot (w_{k+1} - w_{k-1}) + \frac{1}{\alpha^2} \cdot (w_{i-1} + w_{l-1} - w_{i+1} - w_{l+1}) - w_{k+2}\right]$$

$$T_{y,k} = \frac{k}{2 \cdot \Delta y^3} \cdot \left[w_h + 2 \cdot (1+\alpha^2) \cdot (w_l - w_i) + \alpha^2 \cdot (w_{i-1} + w_{i+1} - w_{l-1} - w_{l+1}) - w_m\right]$$

$$\overline{T}_{x,k} = \frac{k}{2 \cdot \Delta x^3} \cdot \left[w_{k-2} + (w_{k+1} - w_{k-1}) \cdot 2 \cdot \left(1 + \frac{2-\nu}{\alpha^2}\right) + \frac{2-\nu}{\alpha^2} \cdot (w_{i-1} + w_{l-1} - w_{i+1} - w_{l+1}) - w_{k+2}\right]$$

$$\overline{T}_{y,k} = \frac{k}{2 \cdot \Delta y^3} \cdot \left[w_h + (w_l - w_i) \cdot 2 \cdot [1 + \alpha^2 \cdot (2-\nu)] + \alpha^2 (2-\nu) \cdot (w_{i+1} + w_{i-1} - w_{l+1} - w_{l-1}) - w_m\right]$$

$\Delta y \neq \Delta x$

$$
\begin{array}{ccccc}
 & & 1/\alpha^2 & & \\
 & 2 & -4\cdot\left(1+1/\alpha^2\right) & 2 & \\
\alpha^2 & -4\cdot\left(1+\alpha^2\right) & 6\cdot\left(\alpha^2+1/\alpha^2\right)+8 & -4\cdot\left(1+\alpha^2\right) & \alpha^2 \\
 & 2 & -4\cdot\left(1+1/\alpha^2\right) & 2 & \\
 & & 1/\alpha^2 & &
\end{array}
$$

$\Delta y = \Delta x$

$$
\begin{array}{ccccc}
 & & 1 & & \\
 & 2 & -8 & 2 & \\
1 & -8 & 20 & -8 & 1 \\
 & 2 & -8 & 2 & \\
 & & 1 & &
\end{array}
$$

Kružne ploče opterećene na savijanje

$$\left(\frac{d^2}{dr^2}+\frac{d}{r\cdot dr}\right)\left(\frac{d^2w}{dr^2}+\frac{dw}{r\cdot dr}\right)=\frac{Z}{k}$$

$$w=w_p+C_1+C_2\cdot\rho^2+C_3\cdot\rho^2\cdot\ln\rho+C_4\cdot\ln\rho \qquad \rho=\frac{r}{a}$$

$$w_p=\frac{1}{k}\cdot\int\frac{dr}{r}\cdot\int r\cdot dr\cdot\int\frac{dr}{r}\cdot\int Z\cdot r\cdot dr$$

$$M_r=-k\left(\frac{d^2w}{dr^2}+\nu\cdot\frac{dw}{r\cdot dr}\right)$$

$$M_\phi=-k\left(\frac{dw}{r\cdot dr}+\nu\cdot\frac{d^2w}{dr^2}\right)$$

$$T_r=-k\cdot\frac{d}{dr}\left(\frac{d^2w}{dr^2}+\frac{dw}{r\cdot dr}\right)=-k\left(\frac{d^3w}{dr^3}+\frac{1}{r}\frac{d^2w}{dr^2}-\frac{1}{r^2}\frac{dw}{dr}\right)$$

Kružne ploče opterećene u svojoj ravni

$$\Delta\Delta F=\left(\frac{d^2}{dr^2}+\frac{d}{r\cdot dr}\right)\left(\frac{d^2F}{dr^2}+\frac{dF}{r\cdot dr}\right)=0$$

$$N_r=\frac{dF}{r\cdot dr} \qquad N_\phi=\frac{d^2F}{dr^2} \qquad N_{r\phi}=0$$

$$F=D+A\cdot\ln r+B\cdot r^2+C\cdot r^2\cdot\ln r$$

$$N_r=\frac{A}{r^2}+2\cdot B+C\cdot(1+2\ln r) \qquad N_\phi=-\frac{A}{r^2}+2\cdot B+C\cdot(3+2\ln r)$$

$$\varepsilon_\varphi=\frac{1}{E\cdot h}\left(N_\varphi-\nu\cdot N_r\right) \qquad \varepsilon_r=\frac{1}{E\cdot h}\left(N_r-\nu\cdot N_\varphi\right)$$

$$u=r\cdot\varepsilon_\varphi=\frac{r}{E\cdot h}\left(N_\varphi-\nu\cdot N_r\right)$$